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The Role of Symbolic Representation in the Cognitive Activity of Young Schoolchildren

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ABSTRACT. In the paper differences between sign-oriented representation and symbolic representation are discussed. The study explored perspectives of using symbolization to teach young schoolchildren the mathematical concept. The author recognizes relevant (that allows the proceeding to sign representation) and irrelevant symbolization. The three groups of young schoolchildren (N=49) were taught the mathematical concept of a function using different programs: traditional, employing sign representation and two experimental – based on relevant and irrelevant symbolizations. The experiment demonstrated that symbolic representation can facilitate mastering of the mathematical concept of a function if the content of symbol possesses structural interrelations that could be converted into sign form.

Keywords: symbol; sign; symbolic image; cognitive development.

Ever since Jean Piaget wrote his seminal works, the phenomenon of symbolic representation has been viewed as children's inherent form of reality reflection, dependent on their individual activity (D. Elkonin, 1978; Piaget, 1945; Vygotsky, 1967). Being an integral part of child psychology, symbolic representation was found to most vividly manifest itself in child play, a traditional subject of developmental psychology. The studies of child play pointed out its similarity to modeling, which occurs when children try to reconstruct the realm of social interrelations (Zaporozhets, 1966).

Symbolic Representation in Play and Schooling

The resemblance between child play and modeling is attributed largely to the fact that both involve substitution of one object by another. However, Sapogova's (1993) study of the substitution occurring in child play and the one occurring in modeling proved their natures to be effectively different in each of these instances. The proof was based on identifying the three key features of modeling:

1. the use of a model by way of its referent
2. the objective correspondence between a model and its referent
3. the use of a model for learning about its referent

The substitution occurring in child play fully complies with the first feature and partially with the second; it contradicts the third key definition of modeling because, for example, "when playing with a stick horse a child can learn nothing new about an actual horse" (Sapogova, 1993, p. 174). Therefore, the substitution occurring in modeling is a form of sign representation, whereas the one employed in child play is symbolic representation. In regard to the learning process, Sapogova postulated the possibility that "in learning activities some children employ not modeling but plain substitution" (p. 179).

The question of what essentially underlies child play—modeling or symbolic representation—was approached by Salmina (1994), who experimentally showed that for young schoolchildren "sign-symbolic tools used in problem-solving don't come as sign systems with their own characteristic traits—i.e., the content doesn't get distinguished from the form of introduction" (p. 74). In other words, the children initially employ symbolic representation, which prevails in child play, rather than sign representation. This result allows us to conclude that whenever children are introduced to models, even to learn sign tools, they initially perceive them as symbols.

A symbol, in contrast to a sign, is essentially affective and is employed when children face difficulties coping with externally imposed problems; however, a symbol is not a tool for solving problems (Piaget, 1945, 1962). This distinction clarifies why preschool and primary school teachers intuitively use symbols in introducing problems—because children's inherent symbolic representation of reality allows them to freely interpret new meanings introduced within the traditional educational process and serves as an orienting tool when they are coping with discrepancies between their own system of meanings and the meanings introduced by educational content.

The use of symbols for mediation of such discrepancies can be found in Ostroverkh's (1998) research with 6-year-olds on the notion of the conservation of matter. During experimental classes children were proclaimed wizards who could animate and transform objects. They learned to make water magical by making it shift shapes when pouring it into variously shaped vessels. Children eagerly accepted this symbolic play. When water in a cup was proclaimed magical (alive) and named Lena, children observed it changing shape by pouring it into different cups, admitting that despite the fact that the water "comes and goes," it remains the same Lena. This symbolization of water proved children's intuitive understanding of the conservation of matter and showed that symbolization can actualize children's cognitive abilities, which are otherwise difficult to engage in the traditional educational process.

Bugrimenko (1994), working with 6-year-old children on a special form of educational process involving symbolization, wrote that "a symbolic character that represents a notion serves, to some extent, in the way a myth does. Its mediating function between a child and an object ... is an introduction into an uncertainty.... The purpose of such sign-symbolic means of education is to arrange the child's actions in-between the two leading activities, playing and learning" (p. 58).

In educational practice, the use of symbolization is most effective in situations where "traditional assistance from a grownup doesn't bring a child to steady self-guidance" (Bugrimenko, 1994, p. 60) or in situations of uncertainty. In my opinion, the orienting within situations of uncertainty can be attributed largely to the key difference in operations with signs and symbols as determined by their properties. Sign representation occurs when the structure of a situation corresponds to the structure of a sign. In this case it channels orientation directly to the meaning of the sign. In contrast, in symbolic representation, as there are no sign tools readily available, orientation is carried out exclusively within the situation's symbolic content, and the meaning is searched for through operations within properties and interrelations of its structure. Being encapsulated within the symbolic content, the operation itself remains symbolic until the structural correspondence between constituent parts of a symbol and an actual situation is discovered; this discovery makes possible the subsequent transference from symbolic to sign mediation and results in the productive resolution of situations of uncertainty. Otherwise, if no adequate structural correspondence between the symbolic and the actual situation is discovered, sign mediation doesn't occur, and the operation remains enclosed within the symbolic content of the situation (Veraksa, 2006; 2011, 2012).

Symbolic Representation in Learning Mathematics

There is a certain similarity between play and learning activities. Play activity not only forms a zone of proximal development but also performs a cognitive function. Through play, and the means it provides, children explore relationships existing in the world of grownups. This is a highly symbolized process because children interpret the objects of their explorations through their own system of meanings.

The same is also true when it comes to learning new mathematical concepts: Children explore them through their own system of natural notions (Vygotsky, 1983). In this case symbolization occurs in the learning process of children, whose natural notions are far from "ideal forms" in their content (Ilienkov, 1984). Assuming that symbolization can be effectively used in the educational process, I chose mathematics for an experiment. Although mathematics is essentially about operating complex structures, some claim that such operations are carried out within the surface, or external (aesthetic), layers of their content.

Sinclair (2004) claims that the aesthetic in mathematics plays several roles, such as an evaluative role (connected with the aesthetic value of mathematical entities such as theorems and proofs, their beauty, elegance, and efficiency); a generative role (a special kind of perception used to generate new ideas and find solutions that otherwise may not be possible to deduce); or a motivational role (connected with the aesthetic response to certain mathematical problems that motivates some people to do mathematics).

Here I would like to highlight the generative role of the aesthetic in mathematics. Poincare (1989) was one of the first mathematicians to see the aesthetic play a major role in the subconscious operations of a mathematician's mind. He argued that the distinguishing feature of a mathematical mind is not logic but aesthetics. From his point of view, the aesthetic principle of harmony serves the mind as a crucial selective function. In other words, an aesthetic response to a problem evokes construction of the necessary cognitive structures that contribute to its solution. Dreyfus and Eisenberg (1986), describing how mathematicians draw upon aesthetic judgments, note that, contrary to widespread opinion, a mathematical mind is connected with the aesthetic. One example of such an opinion is Piaget's theory that views the development of mathematical cognition as the development of a logical structure devoid of intuition and the aesthetic. The works of Papert (1978) and Sinclair (2004) provide insight into the application of the aesthetic for solving algebraic and geometric problems.

I believe that an aesthetic response accompanying problem-solving is natural in situations of uncertainty. It represents inquiry into the external side of a situation in an attempt to orient within it. Play is intrinsically aesthetic, as it tries to fit things together and seeks patterns that connect or integrate. Just like child play, the aesthetic is emotional and symbolic. We find this idea exemplified in Sinclair's work, when she quotes mathematician N. Weiner (1956), who recognized the mathematician's "power to operate with temporary emotional symbols and to organize out of them a semipermanent, recallable language. If one is not able to do this, one is likely to find that his ideas evaporate from the sheer difficulty of preserving them in an as yet unformulated shape" (Sinclair, 2004, p. 272).

Any educational situation children face at school features two main characteristics: an external condition and the culturally introduced rule pertaining to it. For instance, in any mathematical problem there is a set of external conditions, or data, and mathematical rules that define the required set of actions. The uncertainties regarding mathematical problems traditionally pose difficulties for elementary school pupils at various levels—from understanding the initial data and the requirements to the selection and application of the proper rule. However, all these instances can be mediated by symbolic representation.

For this study, I assumed that the uncertainty of a new mathematical problem can be eliminated by symbolic representation as children's orientation within the symbolic content helps them to discern structural interrelations that will be subsequently transferred to actual educational tasks. I chose teaching the mathematical concept of a function, a well-explored subject in various psychological and pedagogical studies (Even, 1990; Vinner & Dreifus, 1989), as the educational task for this study. One of the main difficulties in teaching the mathematical concept of a function is that to learn it young schoolchildren must retain and simultaneously operate two main concepts of a function—it's being an organized set of two types of data (argument and function value) and also a method of transition from one system to another, the principle of which they need to master.

METHOD

Participants

Acquisition of sign systems, including mathematical operations, is highly dependent on the development of overall intellectual abilities. Therefore my colleagues and I used Raven's Coloured Progressive Matrices Test with 49 children (23 boys and 26 girls) from three different third grades in a common Russian public school. In Russia grades 1–4 correspond to primary school. The experiment was organized during the first part of the academic year, when the mean age of the pupils was 9.4.

The children were placed into three categories according to their scores on the test: 24 children with 25–50, 11 children with 50–75, and 14 children with 75–95 percentile frequency scores. These children were evenly distributed among three groups: the experimental group, control group A, and control group B (16, 16, and 17 children, respectively). Therefore each group consisted of three categories of children according to their percentile frequency scores on the test. No statistically significant differences between the test results of girls and boys were found.

DESIGN

The study of the mathematical concept of a function according to the Russian core curriculum for secondary schools begins in the sixth grade (when pupils are approximately 12 years old). The choice to base the experiment on this concept was made after careful research and consultations with math teachers, which indicated that for fourth graders a graph of a function is completely new knowledge lying outside their zone of actual development but at this age children have the necessary background knowledge to master it.

Therefore, for each of the three groups of third graders my colleagues and I developed a 3-week program of six classes of 40 minutes each; the classes covered the mathematical concept of a function and its graphic representation. The experimental classes were designed as standard math classes in a school setting familiar to the children; the classes were conducted by their math teachers. Conformity of classes with the program was supervised by an assistant who was present in each of the classes.

Rather than focusing on the precise definition of a function, the primary task was to make children comprehend the very concept of a function—the idea of the conversion, or the transformation, of the independent variable (argument) into the dependent variable (value).

The three groups of children were given the same initial assignment in the first experimental class and were asked to solve it: "A cyclist travels from point A to point B traveling at 10 km per hour. The distance between the two points is 50 km. Assuming that a bus is traveling at 40 km per hour, how many hours can a bus wait at point A after the cyclist's departure to arrive at point B earlier than the cyclist?"

Not earlier than a week after the last experimental class, all three groups of children took a final test consisting of five problems requiring graphical solutions.

Procedure

Experimental group: relevant symbolization. Based on our previous research, we assumed that introduction of a completely new concept would put children into a situation of uncertainty, where the use of sign tools is hindered. Such uncertainty could trigger the mechanism of symbolic mediation if a relevant image, with a structure resembling the structure of the concept, were introduced.

Therefore in this experiment the first three classes were aimed at introducing a symbolic image that is not directly connected to the mathematical concept in question but intrinsically resembles its structural interrelations; introducing such an image would allow the further transference of the core idea from a symbolic image to the mathematical concept.

The goal of these three preliminary classes was to lead the children to "spontaneously" discover the idea of transformation and to transfer it to the concept of a mathematical function. It was necessary to have a medium representing the concept of transformation that would be easy for a child to operate because of its familiarity.

Since transformation is a core concept of magic, my colleagues and I chose magic as the medium and introduced to the children an image of Fairyland, where two sorceresses can do magic and turn everything into anything. We assumed that children would get emotionally involved in this story, live through it, and imagine being able to turn everything, including themselves, into anything. With help from the teacher the children would inevitably acknowledge the problem that if anyone can be anyone and anything, it is very hard, if not impossible, to tell who is who. The key moment in the story's plot is the initial meeting of the two sorceresses. In this dramatic event, the people of Fairyland cannot tell which sorceress is which; the children were asked for help to solve the problem of telling one sorceress from another, knowing that both of them can become anything they like and can even turn themselves into one another. We assumed that the greatest challenge for the children in this situation would be to reason the grounds on which the two sorceresses could be distinguished from one another. In this situation we expected the play setting to facilitate the search for a solution and to promote critical evaluation of various ideas that might emerge during discussions about who the sorceresses essentially are, what is it that they can transform with their magic, how they can do that, and how it is possible to distinguish one sorceress from another.

In their first class the children tried to solve the introduced initial math assignment to no effect. They seemed lost and unmotivated. Phrases like "we didn't study this," "show us how," "I don't know," "not interesting" were predominant. Children behaved badly, spoke loudly, turned and talked to each other. It was obvious that they couldn't "stick to the task."

When the experimenter began to talk about Fairyland, the situation changed: The children began to listen attentively, were active in sharing their assumptions about the traits of this magical country and its inhabitants, and named a sorceress as one of them. They discussed the ways in which a sorceress is different from an actual human being ("she can cast spells," "can make everything beautiful, magical"). The experimenter then started a discussion about the nature and the purpose of sorcery. The children eventually came to the idea that sorcery is the transformation of objects into something else with the help of a magic wand. The children agreed that a magician is simply a person who can change the surrounding world—that is, transform objects.

In the second class the children were introduced to the problem of how to tell the two sorceresses apart. In the beginning, the children's suggestions were to use their clothes, looks, age, beauty, and other external traits as a basis for distinguishing them. After it became clear that clothes, looks, and everything else can vary and can be changed, the idea emerged of distinguishing them by their names. However, this assumption was also rejected because the sorceresses can respond to each other's names. Then the children proposed that one sorceress could rule over weather and the other one could rule over people and pets. Thus, although they started by analyzing external traits, the children eventually came to the idea that sorceresses might differ in their abilities, meaning that their respective magical domains could be different. For example, some sorceresses have powers over weather; others, over animals; and so on. Children came to the idea that the types of objects that sorceresses have powers over could be the criteria for their differentiation.

In the third class the children picked up where they left the last discussion about the sorceresses and about the ways those two could be told from one another. They remembered a Harry Potter movie and noted that the magic wands of its heroes differed in their magic power rather than visually. They concluded that the magic wands of their sorceresses might also be different in their transformational magic power. The children gave examples of the differences between a strong magic wand and a weak one; one example was the maximum scale a wand could change an object: A strong wand could make an object much bigger than a weak one could. Thus the children intuitively came to the idea of the coefficient of proportionality as a power of a magic wand. Then the children were given time to draw pictures of the sorceresses and the transformation events.

These classes, in our opinion, led the children to understand the relationship between the argument (the initial object) and the rule of its transformation (the power of a magic wand). This relationship characterizes functional dependence.

The second part of the experiment, classes 4 to 6, was dedicated to the transference of ideas from the symbolic image to the actual content of the mathematical concept of a function. In classes 4 and 5 the children were given simple mathematical functions with positive numbers (y=x, y=2x, y=3x) along with their graphs. The experimenter emphasized that, like a sorceress, a function is connected with the transformation of entities. At the last class the children were given problems to solve.

Control group A: irrelevant symbolization. The use of control group A was intended to eliminate factors of emotional involvement and to introduce an additional motivation from the experimental data set.

The first three classes were dedicated to discussions of Fairyland, but for this group the symbolization was structurally irrelevant to the mathematical concept of a function. This time Fairyland was populated by bears, and the children discussed their life in this Fairyland, made up stories about their everyday life. In contrast to the story in the relevant-symbolization group, where the objective characteristics (transformation, for example) of Fairyland were emphasized and brought to the children's attention, in this group the story focused on the subjective side of life in Fairyland, where the emotional traits ("little bears enjoy doing math") of the inhabitants were emphasized.

The first class also began with the same initial mathematical assignment, and likewise it caused confusion and put the children in a situation of uncertainty.

The experimenter presented a story of Fairyland as a place where bears who love to solve difficult assignments live. The children were eager to discuss the traits of this imagined country; they made up stories about bear life and the life of little bears, describing what they like to do and how they spend their time, which games they play.

The second class began with the question "Will we be thinking about bears today?" The children were given the chance to make up stories about the bears' school life, given that the school, just like all of Fairyland, is magical. The experimenter played along and asked questions such as "What subjects are taught in the school where the little bears study?" "What do you think, do all little bears like to go to school in Fairyland?" "Does school in Fairyland differ from an ordinary school?" Then the children were given time to draw their own stories about little bears. During the class, the experimenter emphasized that the little bears enjoy going to school.

In the third class the experimenter focused the discussion on the little bears' attitude toward math in their school. She asked, for example, "Do little bears study math in their school?" "Why do little bears love to study math?"

Classes 4 to 6 were designed in the same way as in the relevant-symbolization group.

Control group B: standard curriculum. The children in control group B were taught the mathematical notion of a function in accordance with the standard national curriculum for sixth graders.

In the first class children learned about the coordinate system. The teachers explained that this system can be found in real life in various forms, like an address, an airplane or theater seat, e-mail, or games like Battleship and chess.

In the next class the teacher explained the correspondence of two numbers to one point in the coordinate system. Examples of finding a point by its coordinates and vice versa were given. The children learned to find the coordinates of the intersection point of two narrow lines and to build a linear graph of two points on a coordinate system.

The third class was dedicated to learning about values. Examples of variable and constant values were discussed (height of a person, height of a room, distance traveled, distance between two points). The domains of variables and of function were explained and exemplified.

In the fourth class the children learned table and letter representation of linear dependence and drew linear graphs for natural numbers.

The children solved problems connected with vehicle motion using graphs of the $y=k{\times}x$ linear function in the fifth and sixth classes.

The control problems (see the Appendix) weren't given to any of the groups. At the last classes children in all the groups were given problems of the same complexity.

The program was designed using the traditional system of meanings employing sign representation.

RESULTS

In the final test a child was given 1 point for each successfully solved problem. The results are presented in Table 1.

Categories of children according to their percentile scores on Raven's Coloured Progressive Matrices Test	Average score (*M*)		
	Experimental Group	Control Group A	Control Group B
Children in the 25–50 percentile	3.38	1.75	1.88
Children in the 50–75 percentile	3.67	3.50	3.75
Children in the 75–95 percentile	4.20	4.00	4.40

Table 1: Average Score on the Final Test for Five Control Problems

The table shows that the 75–95 percentile children of control group B were the most successful in the experimental task (M=4.40). However, the difference in the performance of the 50–75 and the 75–95 percentile children is statistically insignificant. The 25–50 percentile children performed significantly better in the experimental group, as compared with their peers in both control groups. The difference in the average scores of the experimental group compared with the average scores of control group A was 1.50; and compared with the average scores of control group B it was 1.63. Comparison of the average scores between the experimental group and control groups A and B, respectively, on the basis of the Mann-Whitney criterion shows a statistically significant difference on the level of significance of 0.01. The results suggest that in educational practice relevant symbolic representation is beneficial for children who have trouble solving problems that require sign representation.

The results demonstrate that the type of representation employed in educational practice does not make a significant difference for children with high levels of intellectual development (75–95 percentile children). These children operate with both symbolic and sign tools equally effectively and experience no trouble learning, as is proved by their academic success.

The average score of the 50–75 percentile children is naturally lower than that of the 75–95 percentile children. Still, in this category the level of intellectual development remains determinative.

Most important for the hypothesis are the results of the 25–50 percentile children, who were commonly described by their teachers as academic underachievers. These children showed a productive response to the type of representation employed in the educational program. It turned out that a symbol not only was a tool for emotional involvement in the situation but proved to be an effective cognitive instrument.

Thus, we can see that the performance of educational tasks is possible only through sign representation. The high average scores of the children in the 25–50 percentile subgroup are in agreement with this notion for they demonstrate how the children proceeded from uncertainty to symbolic representation, and from symbolic representation to sign representation. Symbolization was purposely used as a tool for bringing the children to develop sign representation, and the 25–50 percentile children did so successfully.

At the same time, comparison of the results within control group A shows that irrelevant symbolization doesn't facilitate the performance of educational tasks. In other words, emotional involvement alone is not enough for proceeding from symbolic to sign representation.

DISCUSSION

A number of works demonstrate the possibility of employing sign tools in preschool educational practice (DeLoache, 2000; Perner, 1993; Poland & van Oers, 2007), ascribing it largely to the double-coding principle—an ability to simultaneously hold two realities (the actual one and the one of a game)—which is acquired through play activities (Lillard, 1993).

It seems important to distinguish between two principal types of situations in which coding occurs. In the first the sum of the meanings of both realities is defined, as, for example, in the DeLoache scale room experiment (DeLoache, 2000); in the second the meanings of one of the realities are relatively uncertain. This second type of situation is the one that constitutes play activity and the application of symbols.

Vygotsky (1967) describes the common instance of a child hopping on a stick imagining riding a horse. In this situation the stick's sum of meanings is derived from its utility, whereas the purport of the child's actions becomes clear only if they are attributed to the sum of the meanings of a horse, which is yet uncertain to the child.

Given that the second type of situation is characterized by uncertainty of the sum of the meanings of one of the realities, the motive for the boy's actions (Leontyev, 2000) is not that he is using the stick because of the unavailability of a real horse but that he is attempting to understand operations with a horse through operations with a stick. The same phenomenon of reality exploration is noted in the works of Diachenko (1987), which demonstrate that children's activity within game reality brings out possibilities that they can later actualize.

The aim in the research described here was to show that symbolic representation in effect differs from sign mediation and has its own area of application within the educational process. The data obtained demonstrate that symbolic representation is closely connected to and occurs in situations where orienting by using sign tools is hindered. This type of representation turns out to be effective in the education of young schoolchildren in cases where the achievement of sign representation is problematic.

Education in general can be viewed as a model of the transition from an ideal form (embodied in concepts and other sign forms of reality representation) to its realization in the real world. In the moment of this transition, as B. Elkonin writes, "a subject of activity emerges ... 'at the point' when the action is required and no pertinent automatism is available, i.e., at the point of form transformation" (1994, p. 32). An ideal form, being a normative product, is internalized by a grownup who translates it to a child. To a child this ideality, even when it is most vivid, remains obscure and uncertain until, according to Leontyev (2000), the child's psyche accommodates to it. This happens when a child's activity collides with the ideal form. Regarding teaching mathematics, Vygotsky wrote, "In a child's development almost always important moments occur; the child's own arithmetic always collides with the other form of arithmetic taught by grownups. Teachers and psychologists must remember that children's internalization of cultural arithmetic always involves conflict" (1983, pp. 202–203). When encountered, an ideal form is spontaneously symbolized by a child because its true meaning is not yet comprehended.

The important task in this research was to establish the possibility of transitioning to sign forms of problem-solving through operations within a symbolic plan. In her work, Sfard (1994, 2000) studied metaphor as a sign-symbolic tool for solving mathematical problems. According to Lakoff and Johnson (1980) a metaphor emerges when parts of some experience are taken as characterizing the similarity between that experience and a new one. Sfard (1994) distinguished two types of mathematical thinking: operational and structural. Structural thinking employs metaphors. Sfard emphasizes that the structure of a metaphor is beyond logical description. Unfortunately, her work doesn't explore the emergence of a symbolic image and regards a metaphor as a defined principle. I believe that a metaphor, being essentially a reduced symbol, is the product of the exploration of a symbolic plan.

CONCLUSION

My research data show that intervention at the symbolization level of a learning process can be effective for children with insufficient sign apparatus.

This finding supports the notion that a symbolic image can be an effective tool for orientation within a situation of uncertainty.

The effective use of symbolization in the educational process depends largely on teacher-child interpersonal interaction - specifically, on the teacher's ability to introduce children to symbols relevant to the studied content. A relevant symbol is a vehicle for the transition to sign representation, and in this role its presence indicates a child's being in the zone of proximal development.

Appendix: Assignments

Problem 1

Four graphs of linear function with different coefficients of proportionality were introduced, each of them representing the motion of a bus, a train, a car, and a motorcycle. Looking at the graphs, a child was supposed to define the motion of the vehicles represented. The children were expected to make their choice based on a graph, rather than on their common notions of the speed of objects.

Problem 2

Four graphs representing bus motion were introduced. Each graph represented a situation in which a bus was at first in motion, then was still, and after that rode again. The children were required to identify the graph that represented a bus that at first rode rapidly, then made a 3-hour stop, and after that rode slowly.

Problem 3

The children were asked to draw a movement graph of a cyclist riding 20 km per hour and to tell how long it took this cyclist to go 35 km.

Problem 4

The children were given the following problem: "A cyclist left from point A for point B going 10 km per hour. Four hours later a bus left point A for point B going 40 km per hour." The children were asked to draw movement graphs of both vehicles and to calculate the distance from point A at which the bus would overtake the cyclist.

Problem 5

A caterpillar started crawling up a tree at the speed of 1 m per hour. It crawled for 4 hours, then stood still for an hour, and then resumed crawling at a speed of 2 m per hour. Two hours after the caterpillar started climbing the tree, a bug began to crawl up the same tree with a speed of 3 m per hour. The children were asked to draw movement graphs of both insects and to find out at what altitude they met.

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Роль символического опосредствования в познавательной деятельности младших школьников

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Аннотация. В статье рассматриваются различия между знаковым и символическим опосредствованием. В исследовании изучается возможность использования символизации для обучения детей младшего школьного возраста математическим понятиям. Автор выделяет релевантную символизацию (которая позволяет перейти к знаковому отражению) и нерелевантную символизацию. Три группы младших школьников (n=49) изучали математическое понятие функции с использованием трех различных программ: традиционной, основанной на знаковом отражении, и двух экспериментальных – основанных на релевантной и нерелевантной символизации. Экспериментальные данные показали, что символическое опосредствование может способствовать овладению математического понятия в случае, когда содержание символа обладает структурным сходством, которое позволяет перевести его в знаковую форму.

Ключевые слова: символ; знак; символический образ; когнитивное развитие.